

લિબર્ટી પેપરસેટ

ધોરણ 12 : ગણિત

Full Solution

સમય : 3 કલાક

અસાઈનમેન્ટ પ્રશ્નપત્ર 2

PART A

1. (C) 2. (D) 3. (A) 4. (B) 5. (C) 6. (B) 7. (D) 8. (C) 9. (C) 10. (A) 11. (B) 12. (B) 13. (D)
14. (D) 15. (A) 16. (A) 17. (C) 18. (B) 19. (B) 20. (C) 21. (A) 22. (C) 23. (A) 24. (D) 25. (B)
26. (A) 27. (B) 28. (C) 29. (D) 30. (B) 31. (A) 32. (C) 33. (A) 34. (C) 35. (B) 36. (C) 37. (D)
38. (A) 39. (B) 40. (D) 41. (D) 42. (C) 43. (B) 44. (A) 45. (B) 46. (C) 47. (A) 48. (D) 49. (C)
50. (D)

PART B

વિભાગ-A

1.

$$\Rightarrow \text{જ.બા.} = \cos^{-1}(4x^3 - 3x)$$

$$x = \cos \theta \text{ ધારો}$$

$$\therefore \theta = \cos^{-1} x, \theta \in [0, \pi]$$

$$\begin{aligned} \therefore \text{જ.બા.} &= \cos^{-1}(4\cos^3 \theta - 3\cos \theta) \\ &= \cos^{-1}(\cos 3\theta) \end{aligned}$$

$$\text{હવે } \frac{1}{2} \leq x \leq 1$$

$$\therefore \cos \frac{\pi}{3} \geq \cos \theta \geq \cos 0$$

$$(\because [0, \frac{\pi}{2}] \text{ માં } \cos \text{ ઘટતું છે})$$

$$\therefore 0 \leq \theta \leq \frac{\pi}{3}$$

$$\therefore 0 \leq 3\theta \leq \pi$$

$$3\theta \in [0, \pi] \quad \dots (1)$$

$$\therefore \text{જ.બા.} = \cos^{-1}(\cos 3\theta)$$

$$= 3\theta$$

$$= 3 \cos^{-1} x$$

$$= \text{SI.બા.}$$

2.

$$\Rightarrow \text{જ.બા.} = \frac{1}{2} \cos^{-1} \left[\frac{1-x}{1+x} \right]$$

$$x = \tan^2 \theta \text{ ધારો.}$$

$$\tan \theta = \sqrt{x}$$

$$\therefore \theta = \tan^{-1} \sqrt{x}, \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2} \right)$$

$$= \frac{1}{2} \cos^{-1} \left[\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} \right]$$

$$= \frac{1}{2} \cdot \cos^{-1}(\cos 2\theta)$$

$$\text{અહીં, } 0 \leq x \leq 1$$

$$\therefore \tan 0 \leq \tan \theta \leq \tan \frac{\pi}{4}$$

$$\therefore 0 \leq \theta \leq \frac{\pi}{4}$$

($\because \tan$ બધા જ ચરણમાં વધતું વિધેય છે.)

$$\therefore 0 \leq 2\theta \leq \frac{\pi}{2}$$

$$2\theta \in \left[0, \frac{\pi}{2} \right] \subset [0, \pi] \quad \dots (1)$$

$$= \frac{1}{2} (2\theta)$$

(\because પરિણામ (1) પરથી)

$$= \theta$$

$$= \tan^{-1} \sqrt{x}$$

$$= \text{SI.બા.}$$

3.

$$\Rightarrow f \text{ એ } x = \frac{\pi}{2} \text{ આગળ સતત છે.}$$

$$\therefore \lim_{x \rightarrow \frac{\pi}{2}} f(x) = f\left(\frac{\pi}{2}\right)$$

$$\text{હવે, } \lim_{x \rightarrow \frac{\pi}{2}} \left(\frac{k \cos x}{\pi - 2x} \right) = 3$$

$$\therefore \lim_{x \rightarrow \frac{\pi}{2}} \frac{k \cos x}{2\left(\frac{\pi}{2} - x\right)} = 3$$

$$\begin{aligned} \therefore \lim_{\frac{\pi}{2}-x \rightarrow 0} \frac{k}{2} \frac{\sin\left(\frac{\pi}{2}-x\right)}{\left(\frac{\pi}{2}-x\right)} &= 3 \left(\begin{array}{l} \because x \rightarrow \frac{\pi}{2} \\ \Rightarrow \frac{\pi}{2}-x \rightarrow 0 \end{array} \right) \\ \therefore \frac{k(1)}{2} &= 3 \\ \therefore k &= 6 \end{aligned}$$

4.

$$\begin{aligned} \Rightarrow I &= \int \frac{dx}{(e^x-1)} \\ &= \int \frac{e^x}{e^x(e^x-1)} dx \end{aligned}$$

→ હવે, $e^x = t$ આદેશ લેતાં,

$$\therefore e^x \cdot dx = dt$$

$$I = \int \frac{dt}{t(t-1)}$$

$$I = \int \frac{t-(t-1)}{t(t-1)} dt$$

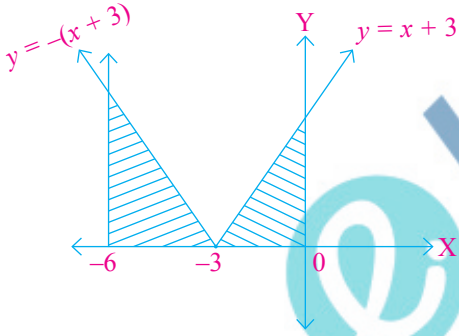
$$= \int \frac{dt}{t-1} - \int \frac{dt}{t}$$

$$= \log|t-1| - \log|t| + c$$

$$I = \log|e^x-1| - \log|e^x| + c$$

$$I = \log\left|\frac{e^x-1}{e^x}\right| + c$$

5.



$$\begin{aligned} \Rightarrow \int_{-6}^0 |x+3| dx &= \int_{-6}^{-3} |x+3| dx + \int_{-3}^0 |x+3| dx \\ &= -\int_{-6}^{-3} (x+3) dx + \int_{-3}^0 (x+3) dx \end{aligned}$$

$$\left(\begin{array}{l} \because -6 < x < -3 \\ \Rightarrow x+3 < 0 \\ \Rightarrow |x+3| = -(x+3) \end{array} \right) \left(\begin{array}{l} \because -3 < x < 0 \\ \Rightarrow x+3 > 0 \\ \Rightarrow |x+3| = x+3 \end{array} \right)$$

$$= -\left[\frac{x^2}{2} + 3x\right]_{-6}^{-3} + \left[\frac{x^2}{2} + 3x\right]_{-3}^0$$

$$= -\left[\left\{\frac{9}{2} - 9\right\} - \left\{\frac{36}{2} - 18\right\}\right]$$

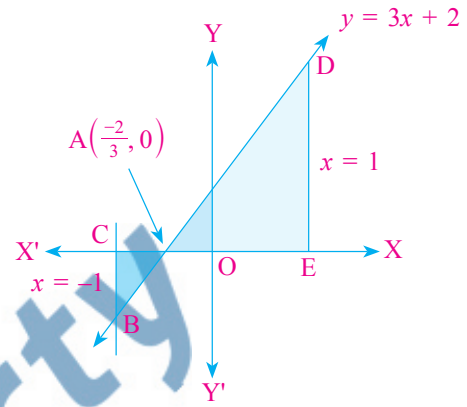
6.

⇒ આકૃતિમાં દર્શાવ્યા પ્રમાણે રેખા $y = 3x + 2$,

X-અક્ષને $\left(-\frac{2}{3}, 0\right)$ માં છેદે છે અને આ

આલેખ $x \in \left(-1, -\frac{2}{3}\right)$ માટે X-અક્ષની નીચે છે અને

આલેખ $x \in \left(-\frac{2}{3}, 1\right)$ માટે X-અક્ષની ઉપર છે.



માંગેલ ક્ષેત્રફળ

= પ્રદેશ ACBAનું ક્ષેત્રફળ + પ્રદેશ ADEAનું ક્ષેત્રફળ

$$= \left| \int_{-1}^{-\frac{2}{3}} (3x+2) dx \right| + \int_{-\frac{2}{3}}^1 (3x+2) dx$$

$$= \left| \left(\frac{3}{2}x^2 + 2x \right)_{-1}^{-\frac{2}{3}} \right| + \left(\frac{3}{2}x^2 + 2x \right)_{-\frac{2}{3}}^1$$

$$= \left| \left(\frac{3}{2} \left(\frac{4}{9} \right) - \frac{4}{3} \right) - \left(\frac{3}{2}(1) + 2(-1) \right) \right| + \left(\frac{3}{2}(1) + 2(1) \right) - \left(\frac{3}{2} \left(\frac{4}{9} \right) + 2 \left(-\frac{2}{3} \right) \right)$$

$$= \left| \frac{2}{3} - \frac{4}{3} - \frac{3}{2} + 2 \right| + \frac{3}{2} + 2 - \frac{2}{3} + \frac{4}{3}$$

$$= \left| \frac{-2}{3} - \frac{3}{2} + 2 \right| + \frac{3}{2} + 2 + \frac{2}{3}$$

$$= \left| \frac{-4-9+12}{6} \right| + \frac{9+12+4}{6}$$

$$= \frac{1}{6} + \frac{25}{6}$$

$$= \frac{26}{6}$$

$$= \frac{13}{3} \text{ ચોરસ એકમ}$$

7.

$$\Rightarrow x \frac{dy}{dx} + 2y = x^2 \log x$$

$$\therefore \frac{dy}{dx} + \frac{2}{x}y = x \log x \quad \dots (1)$$

આ સમીકરણને $\frac{dy}{dx} + P(x)y = Q(x)$ સાથે સરખાવતાં,

$$P(x) = \frac{2}{x}$$

$$Q(x) = x \log x$$

$$\text{સંકલ્પકારક અવયવ; I.F.} = e^{\int P(x) dx}$$

$$= e^{\int \frac{2}{x} dx}$$

$$= e^{2 \log x}$$

$$= e^{\log x^2}$$

$$= x^2$$

પરિણામ (1) ને બંને બાજુ x^2 વડે ગુણતાં,

$$\therefore \frac{dy}{dx} x^2 + 2xy = x^3 \log x$$

$$\therefore \frac{d}{dx}(y x^2) = \int x^3 \log x dx$$

$$\rightarrow u = \log x, v = x^3$$

$$\frac{du}{dx} = \frac{1}{x}$$

$$\therefore y \cdot x^2 = \log x \int x^3 dx - \int \left[\frac{1}{x} \int x^3 dx \right] dx$$

$$= \log x \cdot \frac{x^4}{4} - \int \frac{1}{x} \cdot \frac{x^4}{4} dx$$

$$\therefore y \cdot x^2 = \log x \cdot \frac{x^4}{4} - \frac{x^4}{16} + c$$

$$\therefore y = \log x \cdot \frac{x^2}{4} - \frac{x^2}{16} + cx^{-2}$$

$$\therefore y = \frac{x^2}{16} (4 \log x - 1) + cx^{-2};$$

જે આપેલ વિકલ સમીકરણનો વ્યાપક ઉકેલ છે.

8.

$$\Rightarrow |\vec{a}| = 1, |\vec{b}| = 1, |\vec{c}| = 1$$

$$\text{અહીં, } \vec{a} + \vec{b} + \vec{c} = \vec{0}$$

$$\therefore |\vec{a} + \vec{b} + \vec{c}| = 0$$

$$\therefore |\vec{a} + \vec{b} + \vec{c}|^2 = 0$$

$$\therefore |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2$$

$$+ 2\vec{a} \cdot \vec{b} + 2\vec{b} \cdot \vec{c} + 2\vec{c} \cdot \vec{a} = 0$$

$$\therefore 1 + 1 + 1 + 2$$

$$(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$$

$$\therefore \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = -\frac{3}{2}$$

9.

$$\Rightarrow \text{રેખા } L_1 : \frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$$

$$\vec{r} = (8\hat{i} - 19\hat{j} + 10\hat{k}) + \lambda(3\hat{i} - 16\hat{j} + 7\hat{k})$$

$$\text{રેખાની દિશા } \vec{b}_1 = 3\hat{i} - 16\hat{j} + 7\hat{k}$$

$$\text{રેખા } L_2 : \frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$$

$$\vec{r}_2 = (15\hat{i} + 29\hat{j} + 5\hat{k}) + \lambda(3\hat{i} + 8\hat{j} - 5\hat{k})$$

$$\text{રેખાની દિશા } \vec{b}_2 = 3\hat{i} + 8\hat{j} - 5\hat{k}$$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -16 & 7 \\ 3 & 8 & -5 \end{vmatrix}$$

$$= 24\hat{i} + 36\hat{j} + 72\hat{k}$$

$$= 12(2\hat{i} + 3\hat{j} + 6\hat{k})$$

$$\therefore \text{ માંગેલ રેખાની દિશા } \vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$$

$$\text{રેખા પરનું બિંદુ } A(\vec{a}) = \hat{i} + 2\hat{j} - 4\hat{k}$$

રેખાનું સદિશ સમીકરણ,

$$\therefore \vec{r} = \vec{a} + \lambda \vec{b}, \lambda \in \mathbb{R}$$

$$\therefore \vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$$

10.

$$\Rightarrow \text{ ધારો કે } A(4, 7, 8), B(2, 3, 4),$$

$P(-1, -2, 1), Q(1, 2, 5)$ આપેલ બિંદુઓ છે.

$$\vec{AB} = -2\hat{i} - 4\hat{j} - 4\hat{k}$$

$$\vec{PQ} = 2\hat{i} + 4\hat{j} + 4\hat{k}$$

$$\text{હવે, } \vec{AB} = \lambda \vec{PQ}$$

$$\therefore (-2\hat{i} - 4\hat{j} - 4\hat{k}) = \lambda(2\hat{i} + 4\hat{j} + 4\hat{k}), \lambda \in \mathbb{R}$$

$$\therefore -2 = 2\lambda, -4 = 4\lambda, -4 = 4\lambda$$

$$\therefore \lambda = -1, \lambda = -1, \lambda = -1$$

$$\therefore \vec{AB} \text{ તથા } \vec{PQ} \text{ ના દિઙ્ગુણોત્તર સમાન છે.}$$

$$\therefore \text{ આપેલ બંને રેખાઓ સમાંતર છે.}$$

11.

$$\Rightarrow \text{ બે પાસા ફેંકતા } n = 36$$

$$S = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)\}$$

ઘટના A : બંને પાસા પર ભિન્ન સંખ્યા મળે.

$$\therefore r = 30$$

$$\therefore P(A) = \frac{30}{36} \\ = \frac{5}{6}$$

ઘટના B : પાસા પરની સંખ્યાનો સરવાળો 4 છે.

$$B = \{(1, 3), (2, 2), (3, 1)\}$$

$$A \cap B = \{(1, 3), (3, 1)\}$$

$$\therefore r = 2$$

$$\therefore P(A \cap B) = \frac{2}{36} = \frac{1}{18}$$

$$\therefore P(B | A) = \frac{P(A \cap B)}{P(A)} \\ = \frac{\frac{1}{18}}{\frac{5}{6}} \\ = \frac{1}{15}$$

12.

⇒ A અને B સવાલને સ્વતંત્ર રીતે ઉકેલે તે માટેની

$$\text{સંભાવના } P(A) = \frac{1}{2}$$

$$P(B) = \frac{1}{3}$$

પરસ્પર નિરપેક્ષ ઘટનાઓ છે.

$$\therefore P(A \cap B) = P(A) \cdot P(B)$$

→ કોઈ એકને જ સવાલનો ઉકેલ મળે તેની સંભાવના

$$= P(A \cap B^c) + P(A^c \cap B)$$

$$= P(A) - P(A \cap B) + P(B) - P(A \cap B)$$

$$= P(A) + P(B) - 2P(A)P(B)$$

$$= \frac{1}{2} + \frac{1}{3} - 2 \times \frac{1}{2} \times \frac{1}{3}$$

$$= \frac{1}{2} + \frac{1}{3} - \frac{1}{3}$$

$$= \frac{1}{2}$$

વિભાગ-B

13.

⇒ અહીં $A = \mathbb{R} - \{3\}$, $B = \mathbb{R} - \{1\}$, $f(x) = \left(\frac{x-2}{x-3}\right)$

$$\forall x_1, x_2 \in A, \quad f(x_1) = f(x_2)$$

$$\therefore \frac{x_1-2}{x_1-3} = \frac{x_2-2}{x_2-3}$$

$$\therefore (x_1-2)(x_2-3) = (x_2-2)(x_1-3)$$

$$\therefore x_1x_2 - 3x_1 - 2x_2 + 6 = x_1x_2 - 3x_2 - 2x_1 + 6$$

$$\therefore x_1 = x_2$$

∴ f એ એક-એક વિધેય છે.

ધારો કે, $y \in B = \mathbb{R} - \{1\}$

$$y = f(x)$$

$$\therefore y = \frac{x-2}{x-3}$$

$$\therefore y(x-3) = x-2$$

$$\therefore yx - 3y = x-2$$

$$\therefore yx - x = 3y-2$$

$$\therefore x(y-1) = 3y-2$$

$$\therefore x = \frac{3y-2}{y-1} \in \mathbb{R} - \{3\} \text{ (પ્રદેશ)}$$

$$\text{હવે, } f(x) = f\left(\frac{3y-2}{y-1}\right) = \frac{\frac{3y-2}{y-1}-2}{\frac{3y-2}{y-1}-3} \\ = \frac{3y-2-2y+2}{3y-2-3y+3} = y$$

∴ $\forall y \in B = \mathbb{R} - \{1\}$ માટે

$$x = \frac{3y-2}{y-1} \in A = \mathbb{R} - \{3\} \text{ એવો મળે છે,}$$

કે જેથી $f(x) = y$ થાય છે.

∴ f એ વ્યાપ્ત વિધેય છે.

નોંધ : $f: \mathbb{R} - \left\{-\frac{d}{c}\right\} \rightarrow \mathbb{R} - \left\{\frac{a}{c}\right\}$;

$f(x) = \frac{ax+b}{cx+d}$ એ હંમેશાં એક-એક છે અને વ્યાપ્ત વિધેય છે.

14.

$$\Rightarrow A^T = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

$$\therefore P = \frac{1}{2} (A + A^T)$$

$$= \frac{1}{2} \left\{ \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} + \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} \right\}$$

$$= \frac{1}{2} \begin{bmatrix} 6+6 & -2-2 & 2+2 \\ -2-2 & 3+3 & -1-1 \\ 2+2 & -1-1 & 3+3 \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} 12 & -4 & 4 \\ -4 & 6 & -2 \\ 4 & -2 & 6 \end{bmatrix}$$

$$P = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

$$P^T = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

$$\therefore P = P^T$$

∴ P એ સંમિત શ્રેણિક છે.

$$Q = \frac{1}{2} (A - A^T)$$

$$\begin{aligned}
&= \frac{1}{2} \left\{ \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} - \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} \right\} \\
&= \frac{1}{2} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\
&= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\
\therefore Q^T &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\
\therefore -Q^T &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\
\therefore Q &= -Q^T \\
\therefore Q &\text{ એ વિસંમિત શ્રેણિક છે.} \\
P + Q &= \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\
&= \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} \\
&= A
\end{aligned}$$

15.



આપણે

$$|A| = 1(16 - 9) - 3(4 - 3) + 3(3 - 4) = 1 \neq 0 \text{ મળે છે.}$$

$$\text{હવે, } A_{11} = 7, A_{12} = -1, A_{13} = -1, A_{21} = -3, A_{22} = 1, A_{23} = 0, A_{31} = -3, A_{32} = 0, A_{33} = 1$$

$$\text{માટે } \text{adj } A = \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

$$\begin{aligned}
\text{હવે, } A (\text{adj } A) &= \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} 7-3-3 & -3+3+0 & -3+0+3 \\ 7-4-3 & -3+4+0 & -3+0+3 \\ 7-3-4 & -3+3+0 & -3+0+4 \end{bmatrix} \\
&= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
&= (1) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
&= |A| I
\end{aligned}$$

$$\begin{aligned}
\text{અર્થ, } A^{-1} &= \frac{1}{|A|} \text{adj } A \\
&= \frac{1}{1} \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}
\end{aligned}$$

16.



t ની સાપેક્ષે વિકલન કરતાં,

$$\frac{dx}{dt} = a \left(-\sin t + \frac{1}{\tan \frac{t}{2}} \cdot \sec^2 \frac{t}{2} \cdot \frac{1}{2} \right)$$

$$= a \left(-\sin t + \frac{1}{\frac{\sin \frac{t}{2}}{\cos \frac{t}{2}}} \cdot \frac{1}{\cos^2 \frac{t}{2}} \cdot \frac{1}{2} \right)$$

$$= a \left(-\sin t + \frac{1}{2 \sin \frac{t}{2} \cos \frac{t}{2}} \right)$$

$$= a \left(-\sin t + \frac{1}{\sin t} \right)$$

$$= a \left(\frac{-\sin^2 t + 1}{\sin t} \right)$$

$$= \frac{a \cos^2 t}{\sin t}$$

$$\frac{dy}{dt} = \frac{d}{dt} (a \sin t)$$

$$= a \cos t$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$= \frac{a \cos t}{\frac{a \cos^2 t}{\sin t}}$$

$$= \frac{\sin t}{\cos t}$$

$$\therefore \frac{dy}{dx} = \tan t$$

$$\therefore \frac{d}{dx} \left(\frac{dy}{dx} \right) = \frac{d}{dx} (\tan t)$$

$$= \sec^2 t \frac{dt}{dx}$$

$$= \frac{1}{\cos^2 t} \cdot \frac{1}{\frac{dx}{dt}}$$

$$= \frac{1}{\cos^2 t} \cdot \frac{1}{\frac{a \cos^2 t}{\sin t}}$$

$$= \frac{1}{a} \sec^3 t \cdot \tan t$$

17.



$$y = \frac{4 \sin \theta}{(2 + \cos \theta)} - \theta, \theta \in \left[0, \frac{\pi}{2} \right]$$

$$\therefore \frac{dy}{d\theta} = \frac{(2 + \cos \theta)(4 \cos \theta) - 4 \sin \theta (-\sin \theta)}{(2 + \cos \theta)^2} - 1$$

$$= \frac{8 \cos \theta + 4 \cos^2 \theta + 4 \sin^2 \theta}{(2 + \cos \theta)^2} - 1$$

$$\begin{aligned}
&= \frac{8 \cos \theta + 4(\cos^2 \theta + \sin^2 \theta) - (2 + \cos \theta)^2}{(2 + \cos \theta)^2} \\
&= \frac{8 \cos \theta + 4 - 4 - 4 \cos \theta - \cos^2 \theta}{(2 + \cos \theta)^2} \\
&= \frac{4 \cos \theta - \cos^2 \theta}{(2 + \cos \theta)^2} \\
\frac{dy}{d\theta} &= \frac{\cos \theta (4 - \cos \theta)}{(2 + \cos \theta)^2}
\end{aligned}$$

$$\begin{aligned}
\text{અહીં, } \theta \in \left[0, \frac{\pi}{2}\right] &\Rightarrow \cos \theta \geq 0 \\
&\Rightarrow (4 - \cos \theta) > 0 \\
&\Rightarrow (2 + \cos \theta)^2 > 0 \\
&\Rightarrow \frac{\cos \theta (4 - \cos \theta)}{(2 + \cos \theta)^2} \geq 0 \\
&\Rightarrow \frac{dy}{d\theta} \geq 0
\end{aligned}$$

$\therefore \left[0, \frac{\pi}{2}\right]$ પર f વધતું વિધેય છે.

18.

⇒ અદિશ λ માટે $\vec{\beta}_1 = \lambda \vec{\alpha}$ લો,
 એટલે કે $\vec{\beta}_1 = 3\lambda \vec{i} - \lambda \vec{j}$
 હવે, $\vec{\beta}_2 = \vec{\beta} - \vec{\beta}_1 = (2 - 3\lambda)\vec{i} + (1 + \lambda)\vec{j} - 3\vec{k}$
 તથા $\vec{\beta}_2$ એ $\vec{\alpha}$ ને લંબ હોવાથી,
 આપણી પાસે $\vec{\alpha} \cdot \vec{\beta}_2 = 0$
 એટલે કે $3(2 - 3\lambda) - (1 + \lambda) = 0$ હોવું જોઈએ.
 આથી, $\lambda = \frac{1}{2}$.

$$\text{માટે } \vec{\beta}_1 = \frac{3}{2}\vec{i} - \frac{1}{2}\vec{j} \text{ અને } \vec{\beta}_2 = \frac{1}{2}\vec{i} + \frac{3}{2}\vec{j} - 3\vec{k}$$

19.

⇒ (1) અને (2) ને અનુક્રમે $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ અને
 $\vec{r} = \vec{a}_2 + \mu \vec{b}_2$ સાથે સરખાવતાં, આપણને
 $\vec{a}_1 = \vec{i} + \vec{j}$, $\vec{b}_1 = 2\vec{i} - \vec{j} + \vec{k}$, $\vec{a}_2 = 2\vec{i} + \vec{j} - \vec{k}$
 અને $\vec{b}_2 = 3\vec{i} - 5\vec{j} + 2\vec{k}$ મળે.
 માટે, $\vec{a}_2 - \vec{a}_1 = \vec{i} - \vec{k}$
 અને $\vec{b}_1 \times \vec{b}_2 = (2\vec{i} - \vec{j} + \vec{k}) \times (3\vec{i} - 5\vec{j} + 2\vec{k})$

$$\begin{aligned}
&= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix} \\
&= 3\vec{i} - \vec{j} + 7\vec{k}
\end{aligned}$$

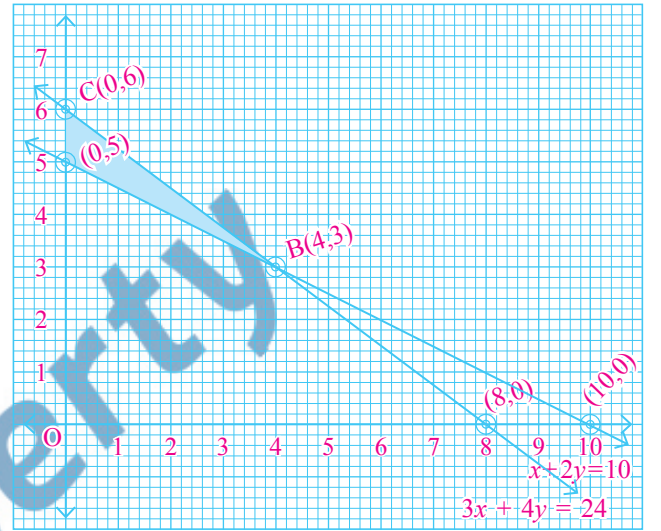
$$\begin{aligned}
\text{તેથી } |\vec{b}_1 \times \vec{b}_2| &= \sqrt{9+1+49} \\
&= \sqrt{59}
\end{aligned}$$

આથી, આપેલી રેખાઓ વચ્ચેનું લઘુતમ અંતર

$$\begin{aligned}
d &= \left| \frac{(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|} \right| \\
&= \left| \frac{3-0+7}{\sqrt{59}} \right| = \frac{10}{\sqrt{59}} \text{ એકમ}
\end{aligned}$$

20.

- ⇒ મર્યાદા સંહતિ (2) થી (4) દ્વારા રચાતો શક્ય ઉકેલનો પ્રથમ પ્રદેશ ABC આકૃતિમાં રંગીન પ્રદેશ તરીકે દર્શાવેલ છે. તે સીમિત છે.
- ⇒ શિરોબિંદુઓ A, B અને C ના ચામ અનુક્રમે (0, 5), (4, 3) અને (0, 6) છે.
- ⇒ હવે, આપણે આ દરેક બિંદુ આગળ Zની કિંમત મેળવીએ.



શક્ય ઉકેલના પ્રદેશનાં શિરોબિંદુઓ	$Z = 200x + 500y$ નું સંગત મૂલ્ય
(0, 5)	2500
(4, 3)	2300 → ન્યૂનતમ
(0, 6)	3000

આમ, બિંદુ (4, 3) આગળ Zનું ન્યૂનતમ મૂલ્ય 2300 મળે છે.

21.

- ⇒ ઘટના E_1 : પસંદ કરેલ દડો પહેલા થેલામાંનો હોય
 ઘટના E_2 : પસંદ કરેલ દડો બીજા થેલામાંનો હોય
 ઘટના A : પસંદ થયેલ બીજો દડો લાલ રંગનો હોય

$$\therefore P(E_1 | A) = \frac{P(E_1) \cdot P(A | E_1)}{P(A)} \text{ (બેયઝ નિયમ)}$$

$$P(E_1) = \frac{1}{2} ; P(E_2) = \frac{1}{2}$$

$$P(A | E_1) = \frac{{}^4C_1}{{}^8C_1} = \frac{4}{8} = \frac{1}{2}$$

$$P(A | E_2) = \frac{{}^2C_1}{{}^8C_1} = \frac{2}{8} = \frac{1}{4}$$

$$\begin{aligned} \therefore P(A) &= P(E_1) \cdot P(A | E_1) + P(E_2) \cdot P(A | E_2) \\ &= \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{4} \\ &= \frac{1}{4} + \frac{1}{8} \\ &= \frac{3}{8} \end{aligned}$$

આમ, પસંદ થયેલ દડો લાલ રંગનો અને તે પ્રથમ થેલામાંથી હોય તેની સંભાવના,

$$\begin{aligned} \therefore P(E_1 | A) &= \frac{P(A | E_1) \cdot P(E_1)}{P(A)} \\ &= \frac{\frac{1}{2} \times \frac{1}{2}}{\frac{3}{8}} \\ &= \frac{2}{3} \end{aligned}$$

વિભાગ-C

22.



$$A^2 = A \cdot A$$

$$= \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0+4 & 0+0+0 & 2+0+6 \\ 0+0+2 & 0+4+0 & 0+2+3 \\ 2+0+6 & 0+0+0 & 4+0+9 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}$$

$$A^3 = A^2 \cdot A$$

$$= \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 5+0+16 & 0+0+0 & 10+0+24 \\ 2+0+10 & 0+8+0 & 4+4+15 \\ 8+0+26 & 0+0+0 & 16+0+39 \end{bmatrix}$$

$$= \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix}$$

$$\text{હવે, SI.બા.} = A^3 - 6A^2 + 7A + 2I$$

$$= \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - 6 \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix} + 7 \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} + 2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{aligned} &= \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} + \begin{bmatrix} -30 & 0 & -48 \\ -12 & -24 & -30 \\ -48 & 0 & -78 \end{bmatrix} + \begin{bmatrix} 7 & 0 & 14 \\ 0 & 14 & 7 \\ 14 & 0 & 21 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 21-30+7+2 & 0+0+0+0 & 34-48+14+0 \\ 12-12+0+0 & 8-24+14+2 & 23-30+7+0 \\ 34-48+14+0 & 0+0+0+0 & 55-78+21+2 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O = \text{જ.બા.} \end{aligned}$$

$$A^3 - 6A^2 + 7A + 2I = 0$$

બંને બાજુ A^{-1} થી ગુણતાં,

$$\therefore A^{-1}(A^3 - 6A^2 + 7A + 2I) = 0 \cdot A^{-1}$$

$$\therefore A^2 - 6A + 7I + 2A^{-1} = 0$$

$$\therefore 2A^{-1} = 6A - A^2 - 7I$$

$$\therefore 2A^{-1} = 6 \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix} - 7 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore 2A^{-1} = \begin{bmatrix} 6 & 0 & 12 \\ 0 & 12 & 6 \\ 12 & 0 & 18 \end{bmatrix} + \begin{bmatrix} -5 & 0 & -8 \\ -2 & -4 & -5 \\ -8 & 0 & -13 \end{bmatrix} + \begin{bmatrix} -7 & 0 & 0 \\ 0 & -7 & 0 \\ 0 & 0 & -7 \end{bmatrix}$$

$$= \begin{bmatrix} -6 & 0 & 4 \\ -2 & 1 & 1 \\ 4 & 0 & -2 \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} -3 & 0 & 2 \\ -1 & \frac{1}{2} & \frac{1}{2} \\ 2 & 0 & -1 \end{bmatrix}$$

23.



આપણે પ્રથમ, બીજી અને ત્રીજી સંખ્યાને અનુક્રમે x , y અને z તરીકે દર્શાવીએ. આપેલ શરતો અનુસાર આપણને

$$x + y + z = 6$$

$$y + 3z = 11$$

$$x + z = 2y \text{ અથવા } x - 2y + z = 0 \text{ મળે.}$$

આ સંહિતિને નીચે પ્રમાણે $AX = B$ સ્વરૂપમાં લખી શકાય :

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ અને } B = \begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$$

અહીં, $|A| = 1(1+6) - 1(0-3) + 1(0-1) = 9 \neq 0$. હવે

આપણે $adjA$ શોધીશું.

$$A_{11} = 1(1+6) = 7, \quad A_{12} = -(0-3) = 3,$$

$$A_{13} = -1, \quad A_{21} = -(1+2) = -3,$$

$$A_{22} = 0, \quad A_{23} = -(-2-1) = 3$$

$$A_{31} = (3-1) = 2, \quad A_{32} = -(3-0) = -3,$$

$$A_{33} = (1-0) = 1$$

$$\text{આથી, } adjA = \begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}$$

$$\text{આથી, } A^{-1} = \frac{1}{|A|} \text{adj}(A) = \frac{1}{9} \begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}$$

$X = A^{-1}B$ હોવાથી

$$X = \frac{1}{9} \begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix} \begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$$

$$\begin{aligned} \text{અથવા } \begin{bmatrix} x \\ y \\ z \end{bmatrix} &= \frac{1}{9} \begin{bmatrix} 42 - 33 + 0 \\ 18 + 0 + 0 \\ -6 + 33 + 0 \end{bmatrix} \\ &= \frac{1}{9} \begin{bmatrix} 9 \\ 18 \\ 27 \end{bmatrix} \\ &= \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \end{aligned}$$

આથી, $x = 1, y = 2, z = 3$.

24.

$$\Rightarrow y = e^{a \cos^{-1} x}$$

હવે, બંને બાજુ x પ્રત્યે વિકલન કરતાં,

$$\frac{dy}{dx} = e^{a \cos^{-1} x} a \cdot \left(\frac{-1}{\sqrt{1-x^2}} \right)$$

$$\therefore \sqrt{1-x^2} \frac{dy}{dx} = -a e^{a \cos^{-1} x}$$

હવે, બંને બાજુ વર્ગ કરતાં,

$$(1-x^2) \left(\frac{dy}{dx} \right)^2 = a^2 [e^{a \cos^{-1} x}]^2$$

$$\therefore (1-x^2) \left(\frac{dy}{dx} \right)^2 = a^2 y^2$$

હવે, બંને બાજુ પુનઃ વિકલન કરતાં,

$$\therefore (1-x^2) 2 \frac{dy}{dx} \frac{d^2 y}{dx^2} + \left(\frac{dy}{dx} \right)^2 (-2x) = a^2 \cdot 2y \frac{dy}{dx}$$

હવે, બંને બાજુ $2 \frac{dy}{dx} \neq 0$ વડે ભાગતાં,

$$\therefore (1-x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} = a^2 y$$

$$\therefore (1-x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} - a^2 y = 0$$

25.

$\Rightarrow r$ ત્રિજ્યાવાળા વર્તુળમાં અંતર્ગત લંબચોરસ $\square ABCD$ છે.

લંબચોરસની લંબાઈ, $AB = CD = x$ ($x \neq 0, x > 0$)

પહોળાઈ, $BC = AD = y$

અહીં, $(2r)^2 = x^2 + y^2$

$$\therefore 4r^2 = x^2 + y^2 \quad \dots \dots \dots (1)$$

હવે, લંબચોરસનું ક્ષેત્રફળ $A =$ લંબાઈ \times પહોળાઈ

$$\therefore A = xy$$

$$\therefore A = x(\sqrt{4r^2 - x^2}) \quad (\because \text{પરિણામ (1)})$$

$\rightarrow f(x) = x^2(4r^2 - x^2)$ લેતાં,

$$f(x) = 4r^2 x^2 - x^4$$

$$\therefore f'(x) = 8r^2 x - 4x^3$$

$$\therefore f''(x) = 8r^2 - 12x^2$$

\rightarrow હવે, મહત્તમ ક્ષેત્રફળ મેળવવા માટે,

$$f'(x) = 0$$

$$\therefore 8r^2 x - 4x^3 = 0$$

$$\therefore 4x(2r^2 - x^2) = 0$$

$$x \neq 0, 2r^2 - x^2 = 0$$

$$\therefore x^2 = 2r^2 \quad \dots \dots \dots (2)$$

હવે $f''(\sqrt{2}r) = 8r^2 - 12x^2$

$$= 8r^2 - 12(2r^2)$$

$$= 8r^2 - 24r^2$$

$$= -16r^2 < 0$$

$\therefore f$ ને મહત્તમ મૂલ્ય મળે.

\rightarrow પરિણામ (1) પરથી,

$$4r^2 = x^2 + y^2$$

$$\therefore 4r^2 = 2r^2 + y^2$$

$$\therefore y^2 = 2r^2$$

$$\therefore y^2 = x^2 \quad (\because \text{પરિણામ (2)})$$

$$\therefore x = y$$

\therefore લંબચોરસ એ ચોરસ છે.

આથી, નિયત વર્તુળમાં અંતર્ગત તમામ લંબચોરસમાં ચોરસનું ક્ષેત્રફળ મહત્તમ હોય છે.

26.

$$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log(1 + \tan x) dx \quad \dots (1)$$

ગુણધર્મ (6) મુજબ, $x = \frac{\pi}{4} - x$

$$I = \int_0^{\frac{\pi}{4}} \log \left(1 + \tan \left(\frac{\pi}{4} - x \right) \right) dx$$

$$= \int_0^{\frac{\pi}{4}} \log \left[1 + \frac{\tan \frac{\pi}{4} - \tan x}{1 + \tan \frac{\pi}{4} \cdot \tan x} \right] dx$$

$$= \int_0^{\frac{\pi}{4}} \log \left(1 + \frac{1 - \tan x}{1 + \tan x} \right) dx$$

$$\begin{aligned}
I &= \int_0^{\frac{\pi}{4}} \log \left(\frac{1 + \tan x + 1 - \tan x}{1 + \tan x} \right) dx \\
&= \int_0^{\frac{\pi}{4}} \log \left(\frac{2}{1 + \tan x} \right) dx \\
&= \int_0^{\frac{\pi}{4}} (\log 2 - \log(1 + \tan x)) dx \\
I &= \log 2 \int_0^{\frac{\pi}{4}} 1 dx - \int_0^{\frac{\pi}{4}} \log(1 + \tan x) dx
\end{aligned}$$

$$I = \log 2 [x]_0^{\frac{\pi}{4}} - I \quad (\because \text{પરિણામ (1) પરથી})$$

$$2I = \log 2 \left(\frac{\pi}{4} - 0 \right)$$

$$\therefore I = \frac{\pi}{8} \log 2$$

27.

⇨ ધારો કે કોઈ t ક્ષણે બેક્ટેરિયાની સંખ્યા p છે.

$$\text{અહીં, } \frac{dp}{dt} \propto p$$

$$\therefore \frac{dp}{dt} = kp \quad (\text{જ્યાં, } k > 0)$$

[જો વિધેય ઘટતું હોય તો $k < 0$ લેવું.]

$$\therefore \frac{dp}{p} = k dt$$

→ બંને બાજુ સંકલન કરતાં,

$$\therefore \int \frac{dp}{p} = k \int 1 dt$$

$$\therefore \log |p| = kt + c \quad \dots (1)$$

→ હવે, $t = 0$ (શરૂઆતમાં) ત્યારે $p = 1,00,000$

$$\therefore \log |1,00,000| = 0 + c$$

$$\therefore c = \log |1,00,000|$$

→ c ની કિંમત પરિણામ (1) માં મૂકતાં,

$$\therefore \log |p| = kt + \log |1,00,000|$$

$$\therefore \log |p| - \log |1,00,000| = kt$$

$$\therefore \log \left| \frac{p}{1,00,000} \right| = kt \quad \dots (2)$$

બે કલાકમાં બેક્ટેરિયા 10% ના દરે વધે છે.

→ $t = 2$ તો $p = 1,00,000 + 1,00,000$ ના 10%

$$\therefore p = 1,00,000 + 1,00,000 \left(\frac{10}{100} \right)$$

$$\therefore p = 1,00,000 + 10,000$$

$$\therefore p = 1,10,000$$

$$\therefore \log \left| \frac{1,10,000}{1,00,000} \right| = 2k$$

$$\therefore 2k = \log \left(\frac{11}{10} \right)$$

$$\therefore k = \frac{1}{2} \log \left(\frac{11}{10} \right)$$

→ k ની કિંમત પરિણામ (2) માં મૂકતાં,

$$\therefore \log \left| \frac{p}{1,00,000} \right| = \frac{1}{2} \log \left(\frac{11}{10} \right) t$$

→ હવે, $p = 2,00,000$ તો $t = ?$

$$\therefore \log \left| \frac{2,00,000}{1,00,000} \right| = \frac{1}{2} \log \left(\frac{11}{10} \right) t$$

$$\therefore \log 2 = \frac{1}{2} \log \left(\frac{11}{10} \right) t$$

$$\therefore 2 \log 2 = \log \left(\frac{11}{10} \right) t$$

$$\therefore t = \frac{2 \log 2}{\log \left(\frac{11}{10} \right)}$$

$$\therefore t = \frac{\log 4}{\log \left(\frac{11}{10} \right)} \text{ કલાક}$$

આથી, $\frac{\log 4}{\log \left(\frac{11}{10} \right)}$ કલાકે બેક્ટેરિયાની સંખ્યા 200000 થશે.